APPROXIMATE METHODS IN ORDER TO DETERMINATE THE DISPLACEMENT FIELDS FOR THE KINEMATICAL ELEMENTS STANDING IN VIBRATION

Dan Gheorghe BAGNARU, Adina CATANEANU
University of Craiova, bagnaru@pcnet.ro

Keywords: kinematical elements, vibrations

Abstract. First of all, using the mechanical model with three concentrated masses, the mathematical model of vibrations in case of a linear-elastic connecting rod, part of a parallelogram mechanism, is presented. Afterwards, the cross displacement field is determined. For a concrete case, the diagrams of cross displacement in relation with time and with distribution of points are given. The knowledge of displacement fields in consequence of the vibrations for the kinematical elements is a preceding stage for the determination of the stress and strain states, useful in machines designing.

1. MATHEMATICAL MODELS OF THE VIBRATIONS FOR A LINEAR-ELASTIC CONNECTING ROD, PART OF A PARALLELOGRAM MECHANISM

In figure 1, the mechanical model with three liberty degrees for the free cross vibrations of the linear-elastic connecting rod OA, part of a R(RRR) mechanism, is given. In [2] and [3], the motion ecuations for a straight linear-elastic kinematical element, obtained by compression of the total mass in "n" points P_i and by using the influence coefficients method, are deducted.

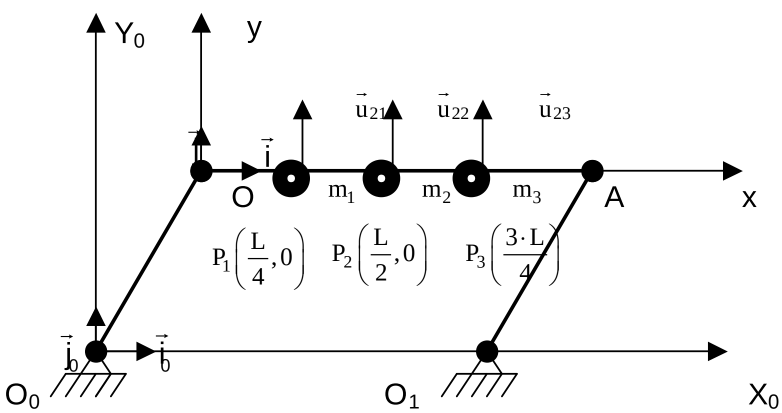

Fig. 1 The mecanical model with three concentrated masses

In figure 1, the total mass m_t of the connecting rod OA is enriched in the points $P_1(L/4,0)$, $P_2(L/2,0)$ and $P_3(3L/4,0)$, thus $m_1 = m_2 = m_3 = \frac{m_t}{3}$.

This way, the mathematical model in approximate method version, that utilizes the mecanical model of the vibrations with finite number degrees, is given under the form:

$$\alpha_{11}m_1\ddot{u}_{21}+\alpha_{12}m_2\ddot{u}_{22}+\alpha_{13}m_3\ddot{u}_{23}+Eu_{21}+a_{02}\left(\alpha_{11}m_1+\alpha_{12}m_2+\alpha_{13}m_3\right)=0$$

$$\alpha_{21}m_1\ddot{u}_{21}+\alpha_{22}m_2\ddot{u}_{22}+\alpha_{23}m_3\ddot{u}_{23}+Eu_{22}+a_{02}\left(\alpha_{21}m_1+\alpha_{22}m_2+\alpha_{23}m_3\right)=0 \qquad (1)$$

$$\alpha_{31}m_1\ddot{u}_{21}+\alpha_{32}m_2\ddot{u}_{22}+\alpha_{33}m_3\ddot{u}_{23}+Eu_{23}+a_{02}\left(\alpha_{31}m_1+\alpha_{32}m_2+\alpha_{33}m_3\right)=0,$$

where:

- $a_{02}=-\omega_0^2 r\sin(\omega_0 t)$ is the acceleration of the point O, end of the OA bar;
- ω_0 is the angular velocity of the O_0O crank;
- ρ is the density of the OA bar ;
- $A=b\cdot h$ is the plan area of the OA bar;
- E is the Young's modulus;
- $I_{zz}=I=\dfrac{bh^3}{12}$ is the geometrical moment of inertia of the plan area of OA bar about the Oz axis;
- b is the width of the plan area of OA bar;
- h is the plan area height of the OA bar;
- $m_t=\rho bhL$ is the total mass of the OA connecting rod;
- r is the O_0O crank length;
- L is the OA connecting rod lenght;

$$\alpha_{11}=\alpha_{33}=\frac{3\cdot L^3}{4\cdot I};\quad \alpha_{12}=\alpha_{23}=\frac{11\cdot L^3}{12\cdot I};\quad \alpha_{22}=\frac{4\cdot L^3}{3\cdot I};\quad \alpha_{13}=\frac{7\cdot L^3}{12\cdot I};$$

$\vec{u}_{21}(t)$, $\vec{u}_{22}(t)$, $\vec{u}_{23}(t)$ are the the cross travels of the points P_1, P_2 and P_3.

2. THE CROSS DISPLACEMENTS FIELD

Applaying the Laplace transform in relation to time to the differential equations system (1), in homogeneous initial conditions, it results an algebraic system that has as unknowns the Laplace's images of the cross displacements. The solution of this system is:

$$u_{21}(s)=u_{23}(s)=\frac{P_1(s)}{P_2(s)};\quad u_{22}(s)=\frac{P_3(s)}{P_2(s)}. \qquad (2)$$

where:

$$P_1(s)=m^2r\omega_0^3\left(-2\alpha_{12}^2+\alpha_{11}\alpha_{22}+\alpha_{13}\alpha_{22}\right)s^2+mrE\omega_0^3\left(\alpha_{11}+\alpha_{12}+\alpha_{13}\right)$$

$$P_2(s)=m^2\left(-2\alpha_{12}^2+\alpha_{11}\alpha_{22}+\alpha_{13}\alpha_{22}\right)s^6+m\Big[E\left(\alpha_{11}+\alpha_{13}+\alpha_{22}\right)+m\omega_0^2\cdot$$

$$\cdot\left(-2\alpha_{12}^2+\alpha_{11}\alpha_{22}+\alpha_{13}\alpha_{22}\right)\Big]s^4+s^2E\left[E+m\omega_0^2\left(\alpha_{11}+\alpha_{13}+\alpha_{22}\right)\right]+E^2\omega_0^2;$$

$$P_3(s)=m^2r\omega_0^3\left(-2\alpha_{12}^2+\alpha_{11}\alpha_{22}+\alpha_{13}\alpha_{22}\right)s^2+mrE\omega_0^3\left(2\alpha_{12}+\alpha_{22}\right)$$

3. GRAPHICAL REPRESENTATION OF THE CROSS DISPLACEMENTS

In (2), applaying the inverse of the Laplace transform, for the concrete case with:

$$L=0,445[m],\ r=0,04[m],\ \omega_0=76,61[s^{-1}],\ b=0,018[m],$$
$$h=0,005[m],\ E=2,1\cdot10^{11}[N/m^2],\ \rho=7800[Kg/m^3]$$

and using the software packages like Mathematica, are obtained the graphical representations from the figures 2, 3 and 4.

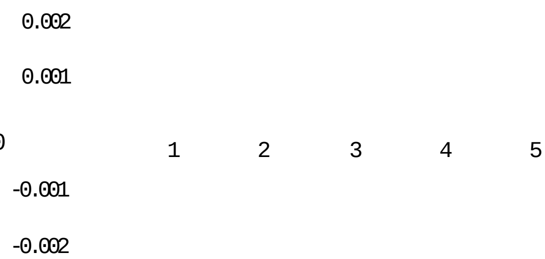

Fig.2 $u_{21}=u_{21}(t)$ ***Fig.3*** $u_{23}=u_{23}(t)$

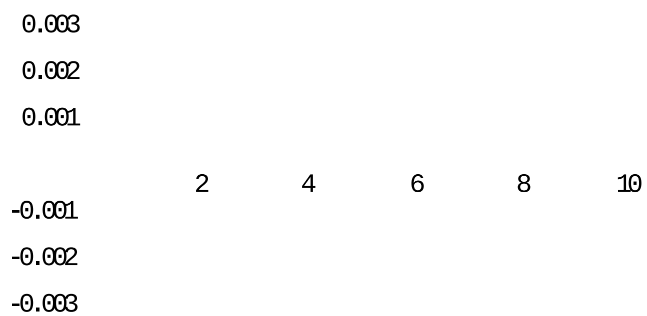

Fig.4 $u_{22}=u_{22}(t)$

4. CONCLUSIONS

In the current practical applications, for a fixed in advance calculus accuracy, it is not unlikely to agree the approximate calculus method. This method presents the advantage of a differential equations system integration, in comparison with the method that supposes a continuous mechanical model and involves the more difficult partial differential ecuations integration.

The knowledge of displacement fields in consequence of the vibrations for the kinematical elements is a preceding stage for the determination of the stress and strain states, useful in machines designing.

BIBLIOGRAPHY

1. Bagnaru, D., Rinderu, P., Vibratiile sistemelor mecanice, Editura LOTUS Craiova, 1998.
2. Bagnaru, D., Ilincioiu, D., Modele matematice ale vibratiilor elementelor cinematice liniar elastice cu un numar finit de grade de libertate, Analele Universitatii din Craiova, Seria Mecanica, 2000, nr.1, pag.46-50.
3. Bagnaru, D., Rizescu, S., Bolcu, D., Vibratiile sistemelor elastice, Editura Didactica si Pedagogica, Bucuresti, 1997, ISBN 973-30-5907-2.
4. Buculei, M., Bagnaru, D., Marghitu, D., Contributii la analiza cinetoelastica a mecanismelor plane pe modele cu un numar finit de grade de libertate, Mec. aplic., nr.2, tom 45, pag.189-196,1986.
5. Ditkine, V., Proudnikov, A., Transformation integrales et calcul operationnel, Ed. Mir,Moscou,1978.